$(7, 2)$-edge-choosability of some 3-regular graphs

Daniel W. Cranston

DIMACS, Rutgers and Bell Labs
dcransto@dimacs.rutgers.edu
Joint with Doug West

$(7,2)$-edge-choosability

Def. A graph is $(7,2)$-edge-choosable if for every assignment of lists of size 7 to its edges, we can choose 2 colors for each edge from its list so that no color is chosen for two incident edges.

$(7, 2)$-edge-choosability

Def. A graph is $(7, 2)$-edge-choosable if for every assignment of lists of size 7 to its edges, we can choose 2 colors for each edge from its list so that no color is chosen for two incident edges.

This differs from standard vertex coloring in 3 ways:

$(7, 2)$-edge-choosability

Def. A graph is $(7, 2)$-edge-choosable if for every assignment of lists of size 7 to its edges, we can choose 2 colors for each edge from its list so that no color is chosen for two incident edges.

This differs from standard vertex coloring in 3 ways:

1. We color from *lists*.

$(7, 2)$-edge-choosability

Def. A graph is $(7, 2)$-edge-choosable if for every assignment of lists of size 7 to its edges, we can choose 2 colors for each edge from its list so that no color is chosen for two incident edges.

This differs from standard vertex coloring in 3 ways:

1. We color from *lists*.
2. Each edge gets 2 colors, rather than 1.

$(7, 2)$-edge-choosability

Def. A graph is $(7, 2)$-edge-choosable if for every assignment of lists of size 7 to its edges, we can choose 2 colors for each edge from its list so that no color is chosen for two incident edges.

This differs from standard vertex coloring in 3 ways:

1. We color from *lists*.
2. Each edge gets 2 colors, rather than 1.
3. We color edges, not vertices.

$(7, 2)$-edge-choosability

Def. A graph is $(7, 2)$-edge-choosable if for every assignment of lists of size 7 to its edges, we can choose 2 colors for each edge from its list so that no color is chosen for two incident edges.

This differs from standard vertex coloring in 3 ways:

1. We color from *lists*.
2. Each edge gets 2 colors, rather than 1.
3. We color edges, not vertices.

Thm. [Haxell-Naserasr] 3-regular graphs are $(7, 2)$-edge-*colorable*.

$(7,2)$-edge-choosability

Def. A graph is $(7,2)$-edge-choosable if for every assignment of lists of size 7 to its edges, we can choose 2 colors for each edge from its list so that no color is chosen for two incident edges.

This differs from standard vertex coloring in 3 ways:

1. We color from *lists*.
2. Each edge gets 2 colors, rather than 1.
3. We color edges, not vertices.

Thm. [Haxell-Naserasr] 3-regular graphs are $(7,2)$-edge-*colorable*.

The biggest open question for edge-choosability is:

Does $\chi'_l(G) = \chi'(G)$ for all graphs G?

$(7, 2)$-edge-choosability

Def. A graph is $(7, 2)$-edge-choosable if for every assignment of lists of size 7 to its edges, we can choose 2 colors for each edge from its list so that no color is chosen for two incident edges.

This differs from standard vertex coloring in 3 ways:

1. We color from *lists*.
2. Each edge gets 2 colors, rather than 1.
3. We color edges, not vertices.

Thm. [Haxell-Naserasr] 3-regular graphs are $(7, 2)$-edge-*colorable*.

The biggest open question for edge-choosability is:

Does $\chi'_l(G) = \chi'(G)$ for all graphs G?

So we examine a similar question for $(r, 2)$-edge-choosability.

But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

But why $(7, 2)$-?

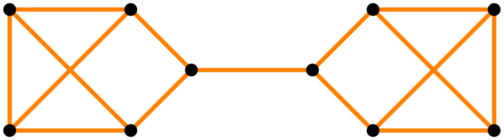
Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

Lem. $r > 6$

But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

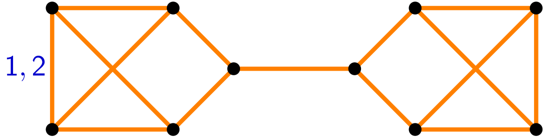
Lem. $r > 6$
Pf.



But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

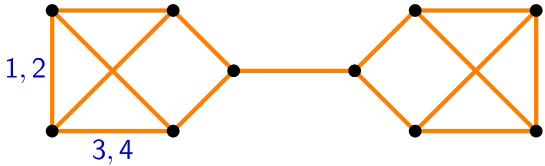
Lem. $r > 6$
Pf.



But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

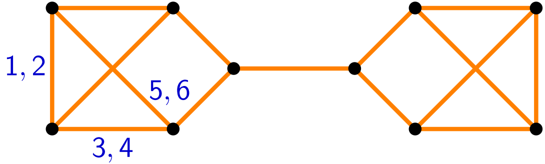
Lem. $r > 6$
Pf.



But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

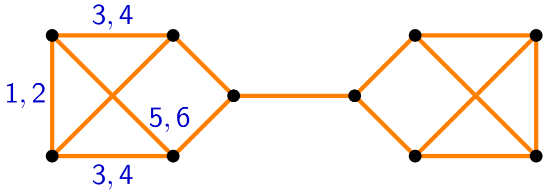
Lem. $r > 6$
Pf.



But why $(7,2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r,2)$-edge-choosble?

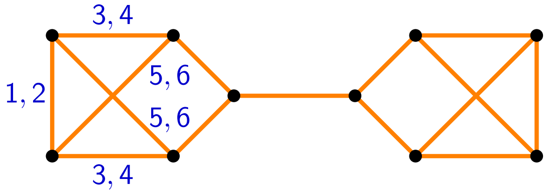
Lem. $r > 6$
Pf.



But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

Lem. $r > 6$
Pf.

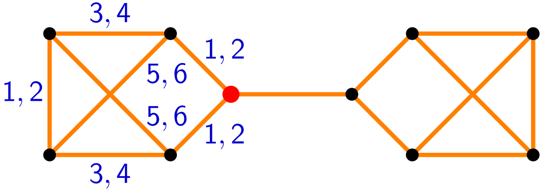


But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

Lem. $r > 6$

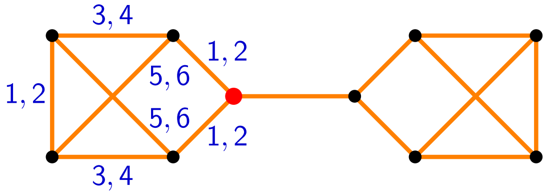
Pf.



But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

Lem. $r > 6$
Pf.

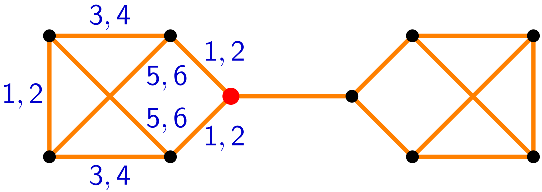


Lem. $r \leq 8$

But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

Lem. $r > 6$
Pf.

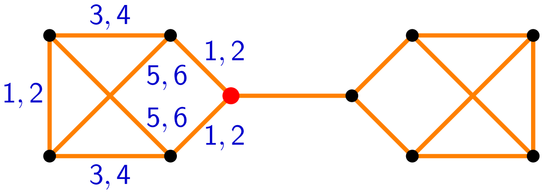


Lem. $r \leq 8$
Pf.

But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

Lem. $r > 6$
Pf.



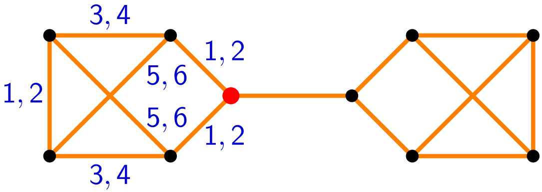
Lem. $r \leq 8$
Pf.

Brooks' Thm:
If G is not a clique or an odd cycle, then $\chi(G) \leq \Delta(G)$.

But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

Lem. $r > 6$
Pf.



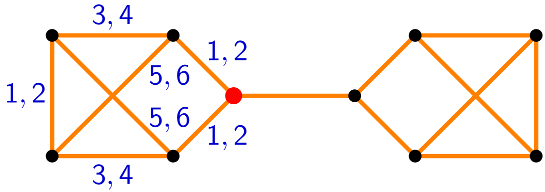
Lem. $r \leq 8$
Pf.

List- Brooks' Thm:
If G is not a clique or an odd cycle, then $\chi_l(G) \leq \Delta(G)$.

But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

Lem. $r > 6$
Pf.



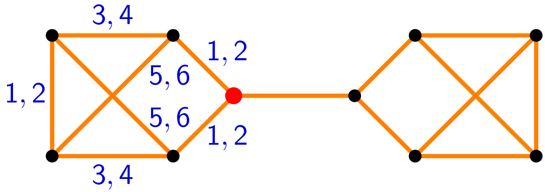
Lem. $r \leq 8$
Pf.

"Super" List- Brooks' Thm:
If G is not a clique or an odd cycle, G is $(m\Delta(G), m)$-choosable.

But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

Lem. $r > 6$
Pf.



Lem. $r \leq 8$
Pf.

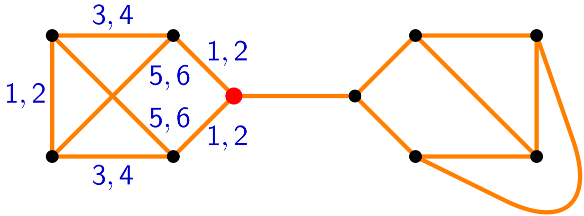
"Super" List- Brooks' Thm:
If G is not a clique or an odd cycle, G is $(m\Delta(G), m)$-choosable.

Consider the line graph $\text{Line}(G)$.

But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

Lem. $r > 6$
Pf.



Lem. $r \leq 8$
Pf.

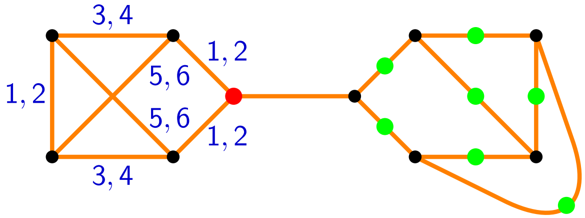
"Super" List- Brooks' Thm:
If G is not a clique or an odd cycle, G is $(m\Delta(G), m)$-choosable.

Consider the line graph $\text{Line}(G)$.

But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

Lem. $r > 6$
Pf.



Lem. $r \leq 8$
Pf.

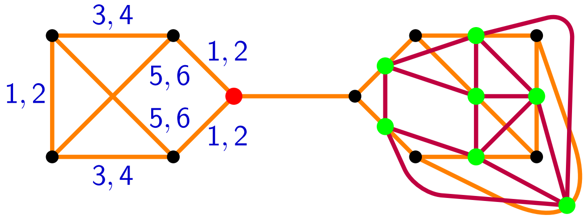
"Super" List- Brooks' Thm:
If G is not a clique or an odd cycle, G is $(m\Delta(G), m)$-choosable.

Consider the line graph Line(G).

But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

Lem. $r > 6$
Pf.



Lem. $r \leq 8$
Pf.

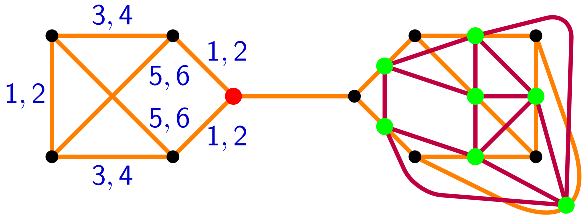
"Super" List- Brooks' Thm:
If G is not a clique or an odd cycle, G is $(m\Delta(G), m)$-choosable.

Consider the line graph Line(G).

But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

Lem. $r > 6$
Pf.



Lem. $r \leq 8$
Pf.

"Super" List- Brooks' Thm:
If G is not a clique or an odd cycle, G is $(m\Delta(G), m)$-choosable.

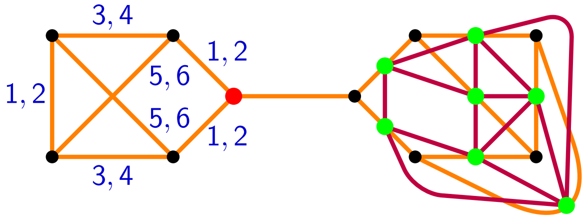
Consider the line graph $\text{Line}(G)$.

$$\Delta(\text{Line}(G)) \leq 4$$

But why $(7, 2)$-?

Ques. [Mohar '03] What is the minimum r such that every 3-regular graph is $(r, 2)$-edge-choosble?

Lem. $r > 6$
Pf.



Lem. $r \leq 8$
Pf.

"Super" List- Brooks' Thm:
If G is not a clique or an odd cycle, G is $(m\Delta(G), m)$-choosable.

Consider the line graph $\text{Line}(G)$.

$$\Delta(\text{Line}(G)) \leq 4$$
$$\implies \text{Line}(G) \text{ is } (8, 2)\text{-choosable}$$

The Main Idea and Two Examples

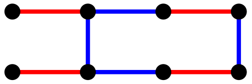
Key Lemma

Let $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ be sets of edges, where A is a matching and b_i is incident to a_i and a_{i+1} but not to any other edges in A (indices mod k). From a d-list assignment L on the edges, we can choose one color at each edge of A so that for each i, together a_i and a_{i+1} receive at most one color from $L(b_i)$.

The Main Idea and Two Examples

Key Lemma

Let $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ be sets of edges, where A is a matching and b_i is incident to a_i and a_{i+1} but not to any other edges in A (indices mod k). From a d-list assignment L on the edges, we can choose one color at each edge of A so that for each i, together a_i and a_{i+1} receive at most one color from $L(b_i)$.

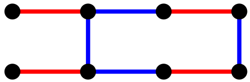


The Main Idea and Two Examples

Key Lemma

Let $A = \{a_1, a_2, \dots, a_k\}$ and $B = \{b_1, b_2, \dots, b_k\}$ be sets of edges, where A is a matching and b_i is incident to a_i and a_{i+1} but not to any other edges in A (indices mod k). From a d-list assignment L on the edges, we can choose one color at each edge of A so that for each i, together a_i and a_{i+1} receive at most one color from $L(b_i)$.

Pf. 1. All lists the same.



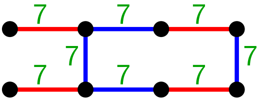
The Main Idea and Two Examples

Key Lemma

Let $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ be sets of edges, where A is a matching and b_i is incident to a_i and a_{i+1} but not to any other edges in A (indices mod k). From a d-list assignment L on the edges, we can choose one color at each edge of A so that for each i, together a_i and a_{i+1} receive at most one color from $L(b_i)$.

Pf. 1. All lists the same.

2. Two adjacent lists differ.

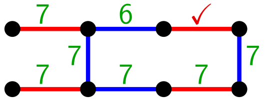


The Main Idea and Two Examples

Key Lemma

Let $A = \{a_1, a_2, \dots, a_k\}$ and $B = \{b_1, b_2, \dots, b_k\}$ be sets of edges, where A is a matching and b_i is incident to a_i and a_{i+1} but not to any other edges in A (indices mod k). From a d-list assignment L on the edges, we can choose one color at each edge of A so that for each i, together a_i and a_{i+1} receive at most one color from $L(b_i)$.

Pf.
1. All lists the same.
2. Two adjacent lists differ.



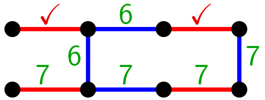
The Main Idea and Two Examples

Key Lemma

Let $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ be sets of edges, where A is a matching and b_i is incident to a_i and a_{i+1} but not to any other edges in A (indices mod k). From a d-list assignment L on the edges, we can choose one color at each edge of A so that for each i, together a_i and a_{i+1} receive at most one color from $L(b_i)$.

Pf. 1. All lists the same.

2. Two adjacent lists differ.

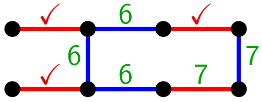


The Main Idea and Two Examples

Key Lemma

Let $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ be sets of edges, where A is a matching and b_i is incident to a_i and a_{i+1} but not to any other edges in A (indices mod k). From a d-list assignment L on the edges, we can choose one color at each edge of A so that for each i, together a_i and a_{i+1} receive at most one color from $L(b_i)$.

Pf.
1. All lists the same.
2. Two adjacent lists differ.

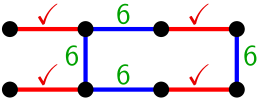


The Main Idea and Two Examples

Key Lemma

Let $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ be sets of edges, where A is a matching and b_i is incident to a_i and a_{i+1} but not to any other edges in A (indices mod k). From a d-list assignment L on the edges, we can choose one color at each edge of A so that for each i, together a_i and a_{i+1} receive at most one color from $L(b_i)$.

Pf. 1. All lists the same.
2. Two adjacent lists differ.

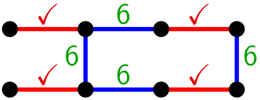


The Main Idea and Two Examples

Key Lemma

Let $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ be sets of edges, where A is a matching and b_i is incident to a_i and a_{i+1} but not to any other edges in A (indices mod k). From a d-list assignment L on the edges, we can choose one color at each edge of A so that for each i, together a_i and a_{i+1} receive at most one color from $L(b_i)$.

Pf. 1. All lists the same.
2. Two adjacent lists differ.



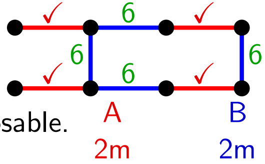
Cor. Even cycles are $(2m, m)$-edge-choosable.

The Main Idea and Two Examples

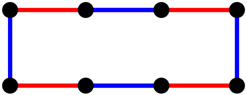
Key Lemma

Let $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ be sets of edges, where A is a matching and b_i is incident to a_i and a_{i+1} but not to any other edges in A (indices mod k). From a d-list assignment L on the edges, we can choose one color at each edge of A so that for each i, together a_i and a_{i+1} receive at most one color from $L(b_i)$.

Pf. 1. All lists the same.
2. Two adjacent lists differ.



Cor. Even cycles are $(2m, m)$-edge-choosable.

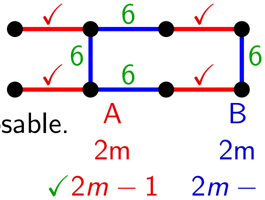


The Main Idea and Two Examples

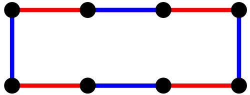
Key Lemma

Let $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ be sets of edges, where A is a matching and b_i is incident to a_i and a_{i+1} but not to any other edges in A (indices mod k). From a d-list assignment L on the edges, we can choose one color at each edge of A so that for each i, together a_i and a_{i+1} receive at most one color from $L(b_i)$.

Pf.
1. All lists the same.
2. Two adjacent lists differ.



Cor. Even cycles are $(2m, m)$-edge-choosable.

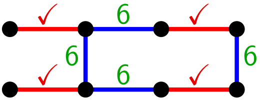


The Main Idea and Two Examples

Key Lemma

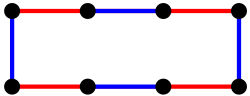
Let $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ be sets of edges, where A is a matching and b_i is incident to a_i and a_{i+1} but not to any other edges in A (indices mod k). From a d-list assignment L on the edges, we can choose one color at each edge of A so that for each i, together a_i and a_{i+1} receive at most one color from $L(b_i)$.

Pf. 1. All lists the same.
2. Two adjacent lists differ.



A	B
2m	2m
✓ $2m-1$	$2m-1$
✓ $2m-2$	$2m-2$ ✓

Cor. Even cycles are $(2m, m)$-edge-choosable.

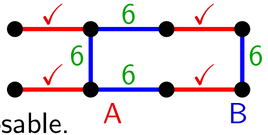


The Main Idea and Two Examples

Key Lemma

Let $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ be sets of edges, where A is a matching and b_i is incident to a_i and a_{i+1} but not to any other edges in A (indices mod k). From a d-list assignment L on the edges, we can choose one color at each edge of A so that for each i, together a_i and a_{i+1} receive at most one color from $L(b_i)$.

Pf. 1. All lists the same.
2. Two adjacent lists differ.

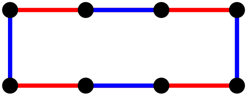


2m 2m

✓$2m-1$ $2m-1$

✓$2m-2$ $2m-2$✓

Cor. Even cycles are $(2m, m)$-edge-choosable.



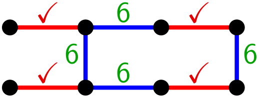
Cor. 3-edge-colorable graphs are $(7, 2)$-edge-choosable.

The Main Idea and Two Examples

Key Lemma

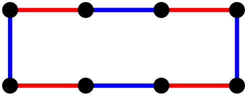
Let $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ be sets of edges, where A is a matching and b_i is incident to a_i and a_{i+1} but not to any other edges in A (indices mod k). From a d-list assignment L on the edges, we can choose one color at each edge of A so that for each i, together a_i and a_{i+1} receive at most one color from $L(b_i)$.

Pf. 1. All lists the same.
2. Two adjacent lists differ.



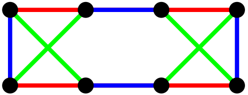
A	B
2m	2m
✓$2m-1$	$2m-1$
✓$2m-2$	$2m-2$✓

Cor. Even cycles are $(2m, m)$-edge-choosable.



Cor. 3-edge-colorable graphs are $(7, 2)$-edge-choosable.

A	B	C
7	7	7

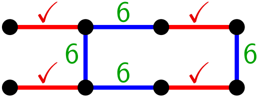


The Main Idea and Two Examples

Key Lemma

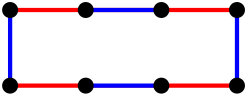
Let $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ be sets of edges, where A is a matching and b_i is incident to a_i and a_{i+1} but not to any other edges in A (indices mod k). From a d-list assignment L on the edges, we can choose one color at each edge of A so that for each i, together a_i and a_{i+1} receive at most one color from $L(b_i)$.

Pf.
1. All lists the same.
2. Two adjacent lists differ.



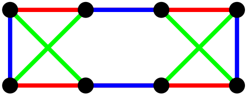
A	B
2m	2m
✓$2m-1$	$2m-1$
✓$2m-2$	$2m-2$✓

Cor. Even cycles are $(2m, m)$-edge-choosable.



Cor. 3-edge-colorable graphs are $(7, 2)$-edge-choosable.

A	B	C
7	7	7
✓6	6	5



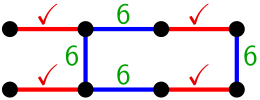
The Main Idea and Two Examples

Key Lemma

Let $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ be sets of edges, where A is a matching and b_i is incident to a_i and a_{i+1} but not to any other edges in A (indices mod k). From a d-list assignment L on the edges, we can choose one color at each edge of A so that for each i, together a_i and a_{i+1} receive at most one color from $L(b_i)$.

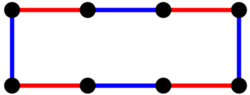
Pf. 1. All lists the same.

2. Two adjacent lists differ.



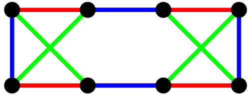
A	B
2m	2m
✓ $2m-1$	$2m-1$
✓ $2m-2$	$2m-2$ ✓

Cor. Even cycles are $(2m, m)$-edge-choosable.



Cor. 3-edge-colorable graphs are $(7, 2)$-edge-choosable.

A	B	C
7	7	7
✓6	6	5
✓5	6	5

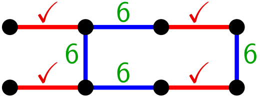


The Main Idea and Two Examples

Key Lemma

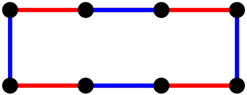
Let $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ be sets of edges, where A is a matching and b_i is incident to a_i and a_{i+1} but not to any other edges in A (indices mod k). From a d-list assignment L on the edges, we can choose one color at each edge of A so that for each i, together a_i and a_{i+1} receive at most one color from $L(b_i)$.

Pf. 1. All lists the same.
2. Two adjacent lists differ.

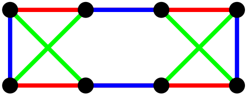


A	B
2m	2m
✓$2m-1$	$2m-1$
✓$2m-2$	$2m-2$✓

Cor. Even cycles are $(2m, m)$-edge-choosable.



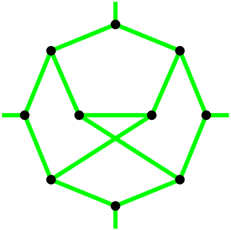
Cor. 3-edge-colorable graphs are $(7, 2)$-edge-choosable.



A	B	C
7	7	7
✓6	6	5
✓5	6	5
✓✓4	4	4

But wait, there's more. . .

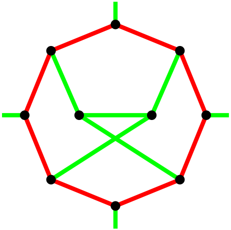
Ex.



Petersen Graph

But wait, there's more. . .

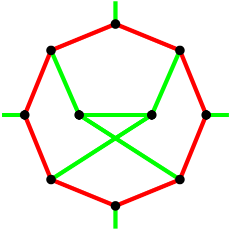
Ex.



Petersen Graph

But wait, there's more...

Ex.

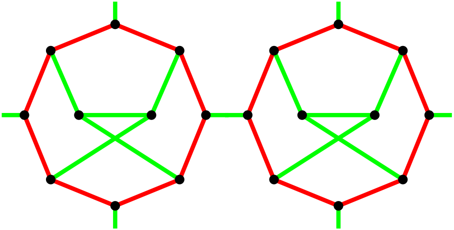


Petersen Graph

Def. A MED decomposition of a 3-regular graph is a decomposition into subgraphs G_1, G_2, and G_3, where G_1 is a Matching, the components of G_2 are Even cycles, and G_3 consists of independent Double-stars.

But wait, there's more...

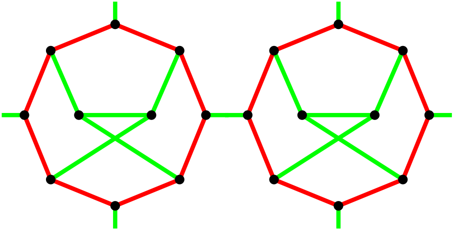
Ex.



Def. A MED decomposition of a 3-regular graph is a decomposition into subgraphs G_1, G_2, and G_3, where G_1 is a Matching, the components of G_2 are Even cycles, and G_3 consists of independent Double-stars.

But wait, there's more...

Ex.



Def. A MED decomposition of a 3-regular graph is a decomposition into subgraphs G_1, G_2, and G_3, where G_1 is a Matching, the components of G_2 are Even cycles, and G_3 consists of independent Double-stars.

Thm. Every 3-regular graph G that has a MED decomposition is $(7, 2)$-edge-choosable.

Proof sketch

Thm. Every 3-regular graph G that has a MED decomposition is $(7, 2)$-edge-choosable.

Proof sketch

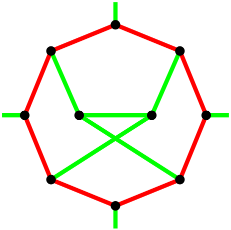
Thm. Every 3-regular graph G that has a MED decomposition is $(7, 2)$-edge-choosable.

Pf. Sketch Choose 2 colors for each edge in the Matching or a Double-star, so that each edge in an Even cycle has 4 colors left. Now we finish, since Even cycles are $(4, 2)$-edge-choosable.

Proof sketch

Thm. Every 3-regular graph G that has a MED decomposition is $(7, 2)$-edge-choosable.

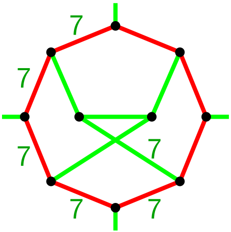
Pf. Sketch Choose 2 colors for each edge in the Matching or a Double-star, so that each edge in an Even cycle has 4 colors left. Now we finish, since Even cycles are $(4, 2)$-edge-choosable.



Proof sketch

Thm. Every 3-regular graph G that has a MED decomposition is $(7, 2)$-edge-choosable.

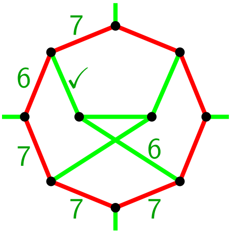
Pf. Sketch Choose 2 colors for each edge in the Matching or a Double-star, so that each edge in an Even cycle has 4 colors left. Now we finish, since Even cycles are $(4, 2)$-edge-choosable.



Proof sketch

Thm. Every 3-regular graph G that has a MED decomposition is $(7,2)$-edge-choosable.

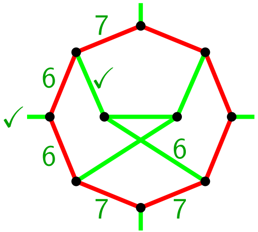
Pf. Sketch Choose 2 colors for each edge in the Matching or a Double-star, so that each edge in an Even cycle has 4 colors left. Now we finish, since Even cycles are $(4,2)$-edge-choosable.



Proof sketch

Thm. Every 3-regular graph G that has a MED decomposition is $(7, 2)$-edge-choosable.

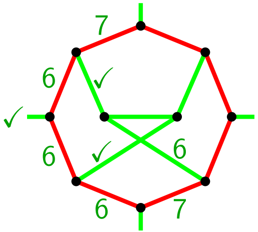
Pf. Sketch Choose 2 colors for each edge in the Matching or a Double-star, so that each edge in an Even cycle has 4 colors left. Now we finish, since Even cycles are $(4, 2)$-edge-choosable.



Proof sketch

Thm. Every 3-regular graph G that has a MED decomposition is $(7, 2)$-edge-choosable.

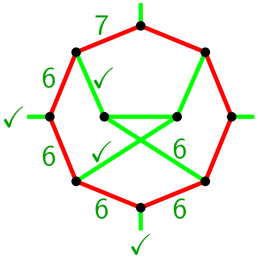
Pf. Sketch Choose 2 colors for each edge in the Matching or a Double-star, so that each edge in an Even cycle has 4 colors left. Now we finish, since Even cycles are $(4, 2)$-edge-choosable.



Proof sketch

Thm. Every 3-regular graph G that has a MED decomposition is $(7, 2)$-edge-choosable.

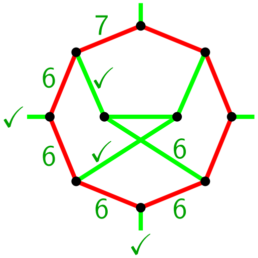
Pf. Sketch Choose 2 colors for each edge in the Matching or a Double-star, so that each edge in an Even cycle has 4 colors left. Now we finish, since Even cycles are $(4, 2)$-edge-choosable.



Proof sketch

Thm. Every 3-regular graph G that has a MED decomposition is $(7,2)$-edge-choosable.

Pf. Sketch Choose 2 colors for each edge in the Matching or a Double-star, so that each edge in an Even cycle has 4 colors left. Now we finish, since Even cycles are $(4,2)$-edge-choosable.



Conj. Every 2-connected 3-regular graph has a MED decomposition.