Acyclic Edge-coloring of Planar Graphs

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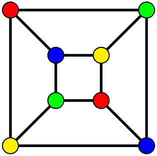

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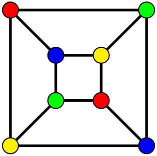

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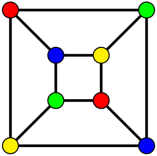

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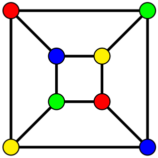

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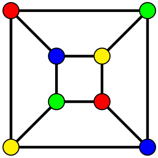

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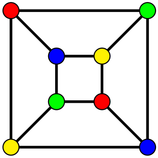

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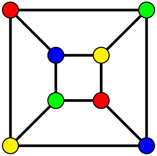

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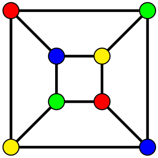

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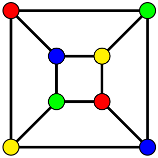

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Thm: If G is planar, then $\chi'_a(G) \leq \Delta + 25$. (C–H–M '09)...
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Main Thm: Conjecture 2 is true

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Main Thm: Conjecture 2 is true; $\Delta_0 = 4.2 * 10^{14}$ suffices.

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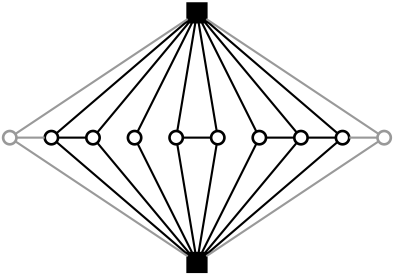

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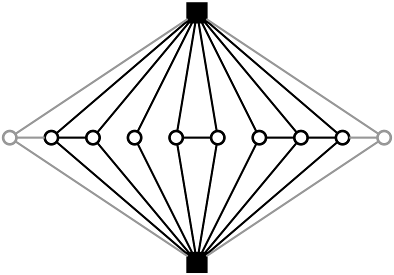

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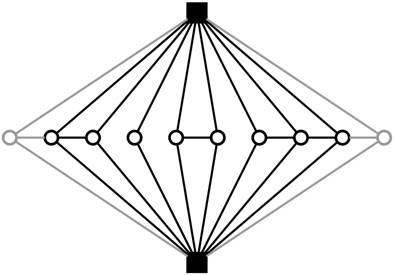

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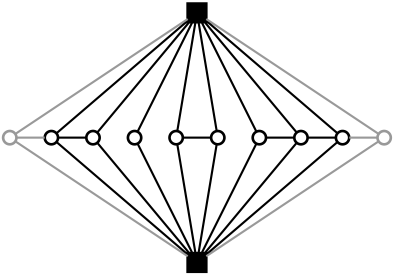

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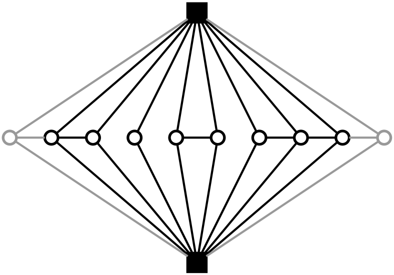

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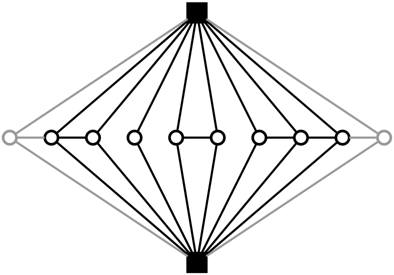

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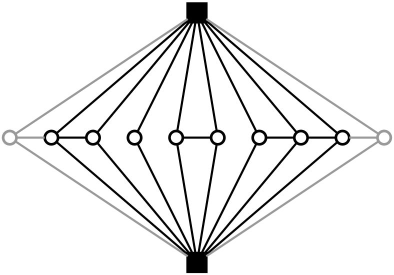

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(RC3) big vertex v with ≤ 35 nbrs not its child in a bunch; or

(RC4) very big vertex v with ≤ 141415 nbrs not child in a bunch.

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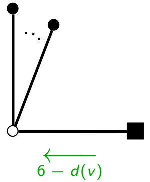

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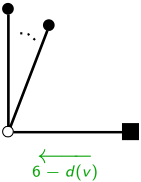

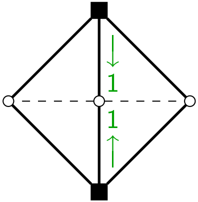

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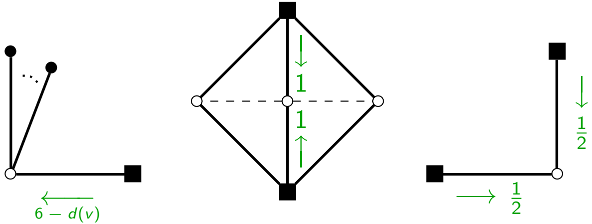

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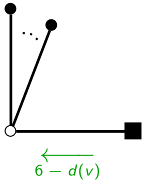

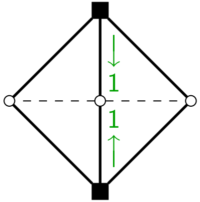

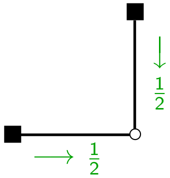

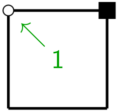

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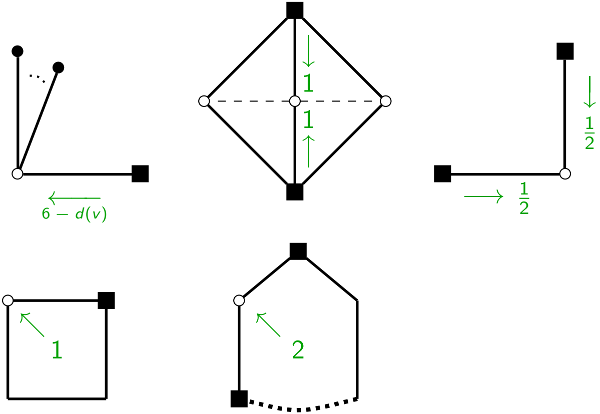

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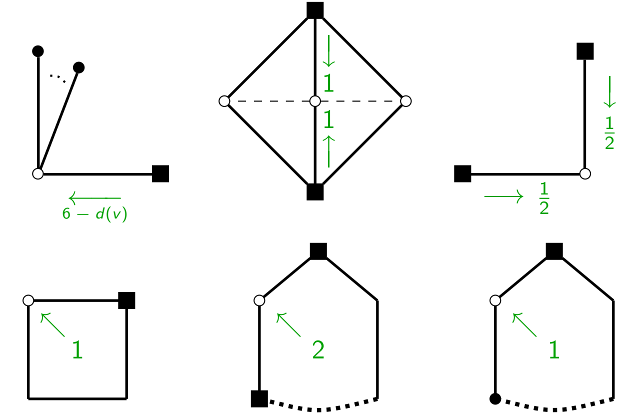

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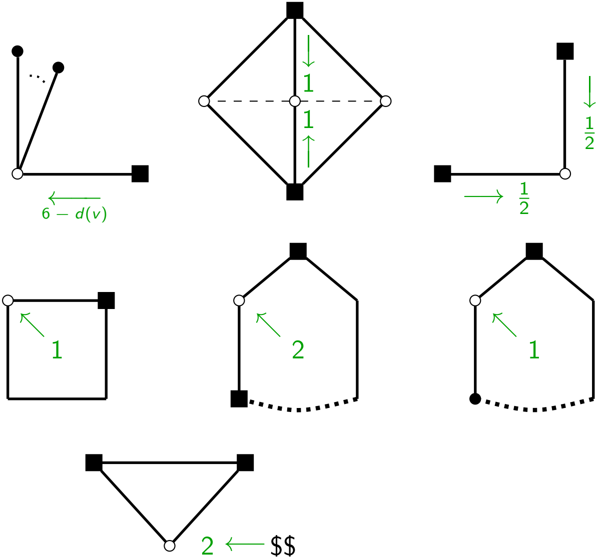

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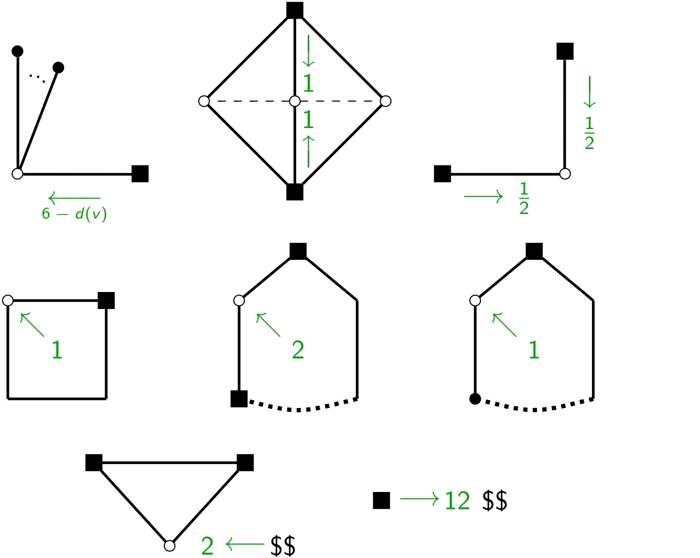

Reducibility Sketch

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Lem: Fix $q \geq 100$. G has no v with $d(v) - \Delta + |\mathcal{W}| \geq q + \sqrt{5q}$, where $\mathcal{W}$ is 5^--neighbors w of v with $\sum_{x \in N(w) \setminus v} d(x) \leq q$.

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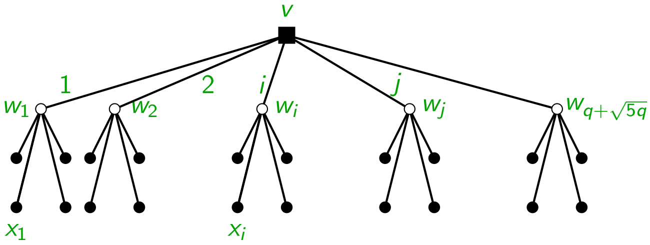

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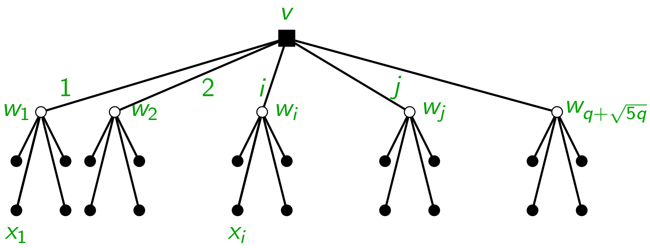

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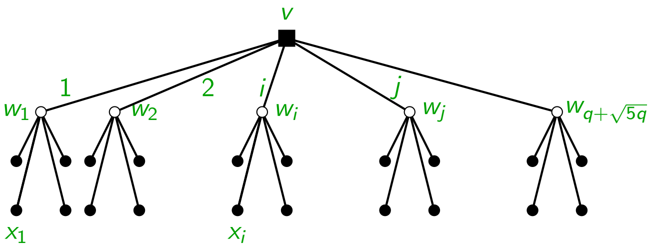

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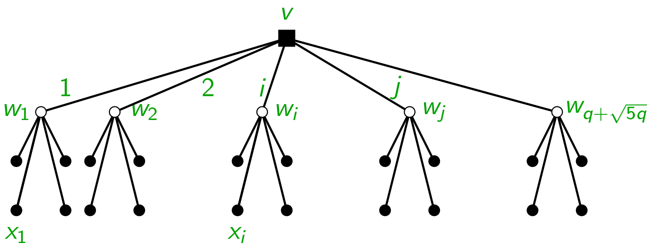

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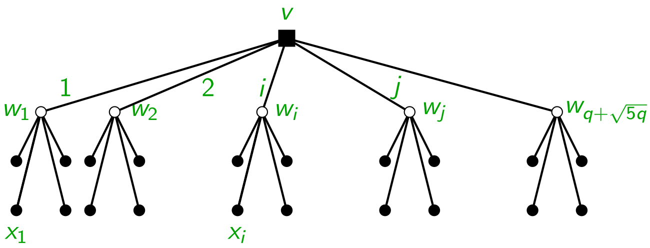

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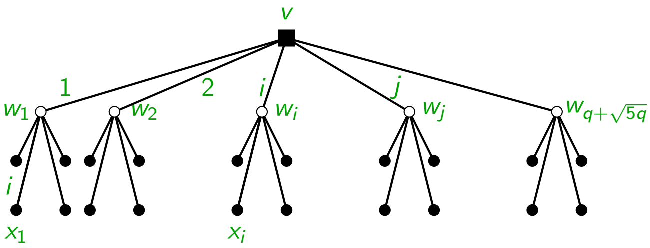

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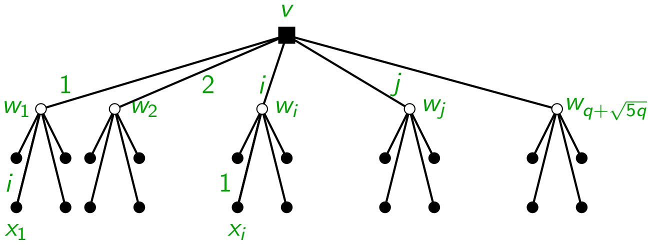

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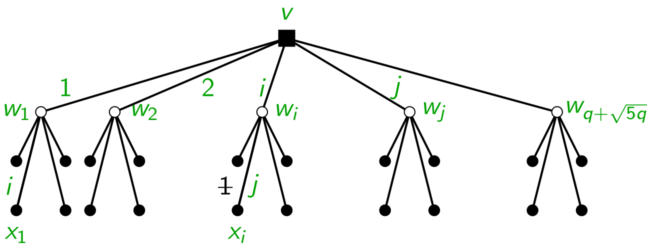

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