Regular Graphs are Antimagic

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Joint with Yu-Chang Liang and Xuding Zhu

Slides available on my webpage

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9 November 2015

Magic Squares

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62	4	13	51	46	20	29	35
5	59	54	12	21	43	38	28
52	14	3	61	36	30	19	45
11	53	60	6	27	37	44	22
64	2	15	49	48	18	31	33
7	57	56	10	23	41	40	26
50	16	1	63	34	32	17	47
9	55	58	8	25	39	42	24

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12	51	56	31	10	33	38	29
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16 3 2 13
5 10 11 8
9 6 7 12
4 15 14 1


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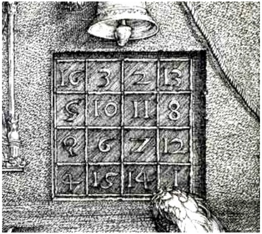
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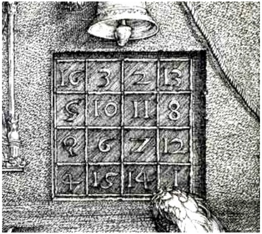

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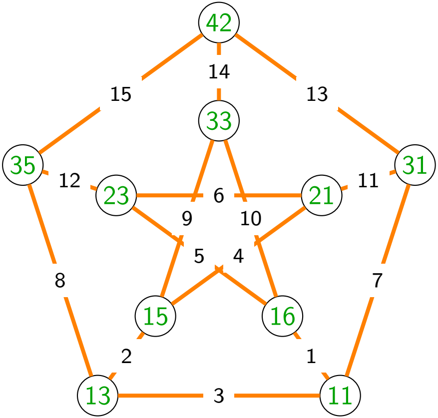

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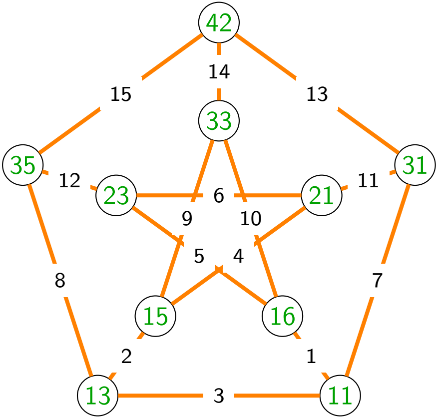

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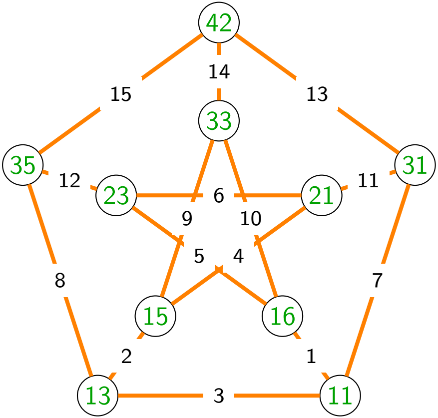

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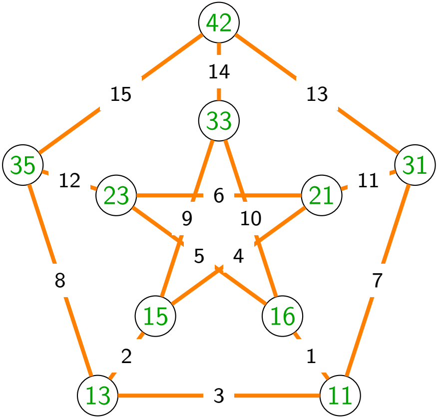

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Cor. This would imply both Hartsfield–Ringel conjectures.

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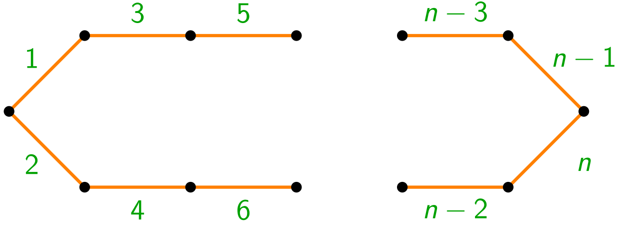

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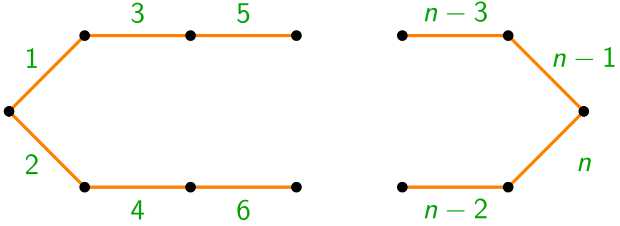

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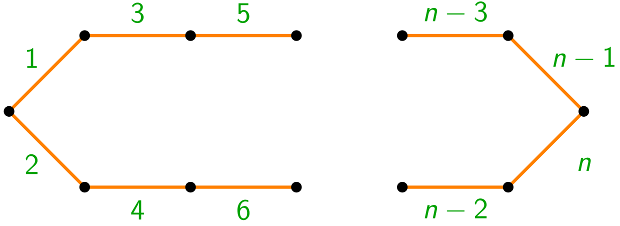

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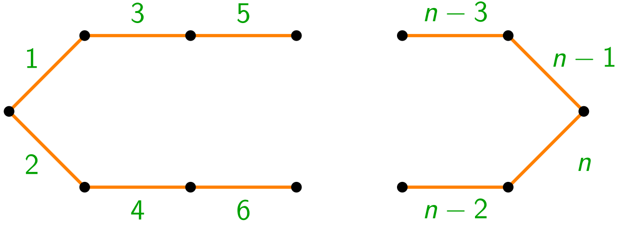

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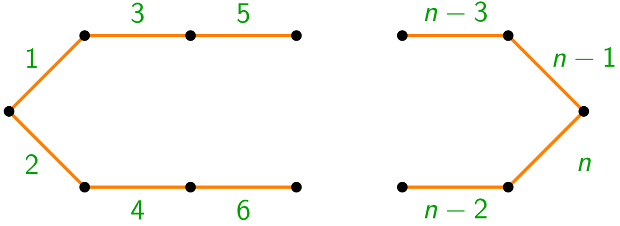

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Cor. Just need to check $f(v_1) \neq f(v_2)$ when $v_1, v_2 \in D_i \cup D_{i-1}$.

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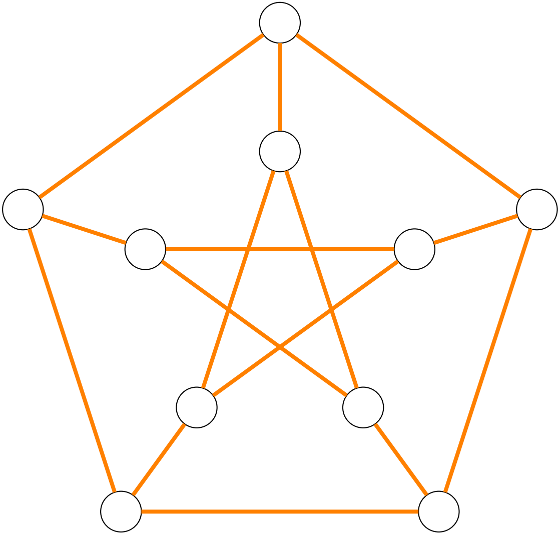

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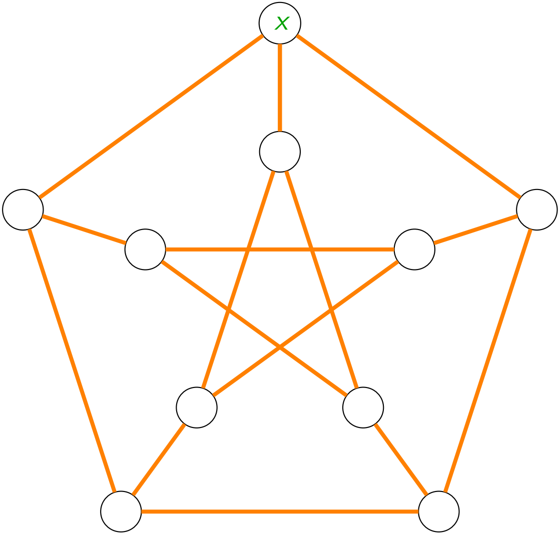

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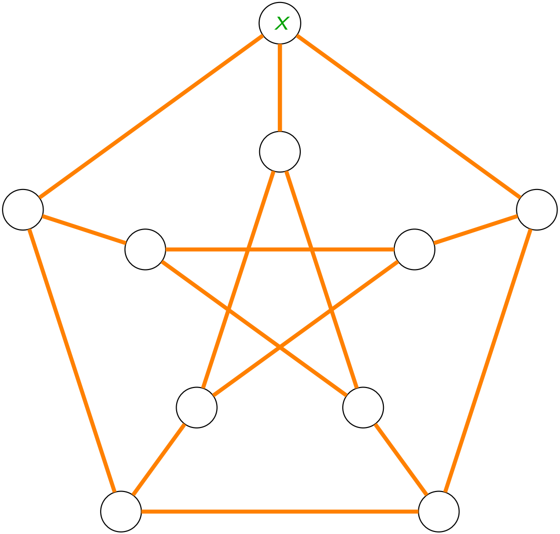

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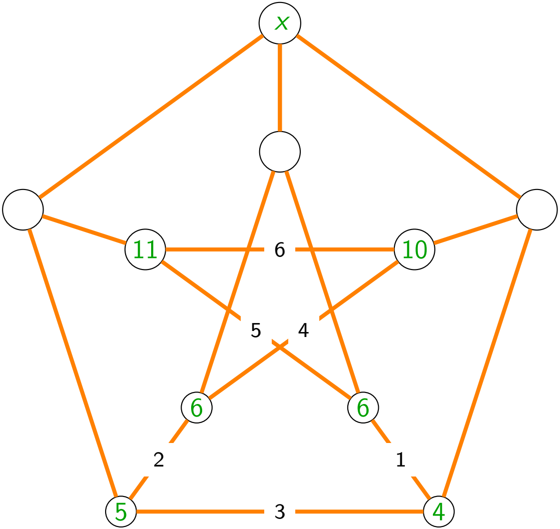

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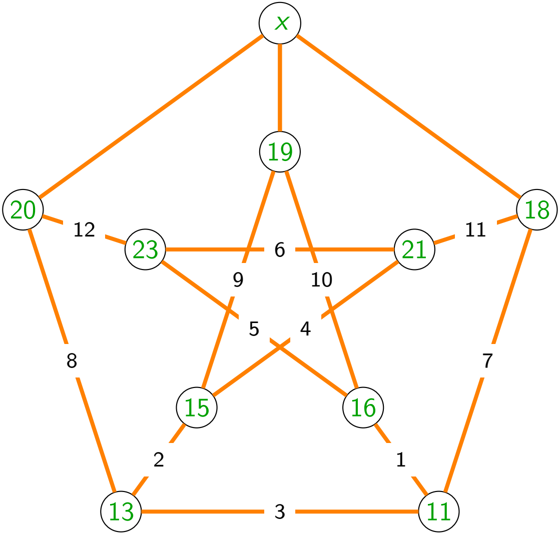

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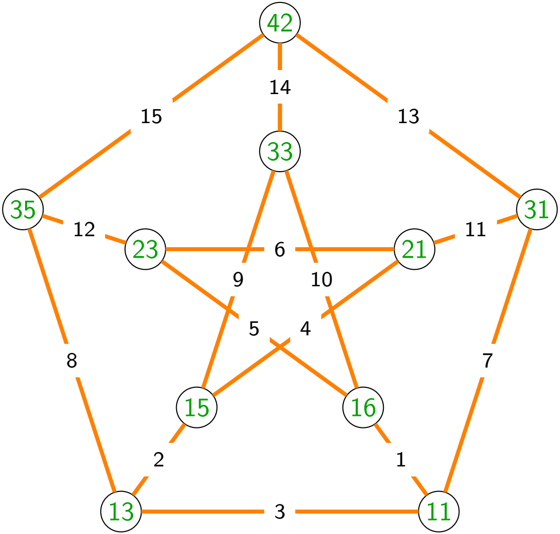

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Pf idea
Edges induce a bipartite graph; each vertex has degree at most 2.

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