List colorings of K_5-minor-free graphs with special list assignments

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Cycles and Colourings
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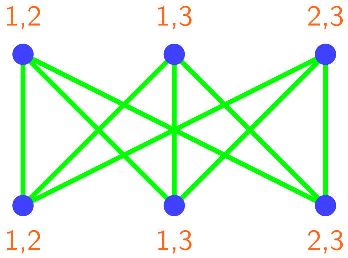

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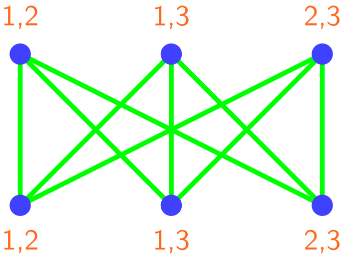

So, $\chi_l(K_{3,3}) > 2 = \chi(K_{3,3})$.

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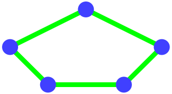
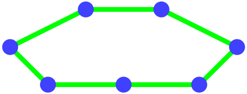
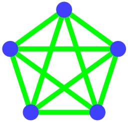

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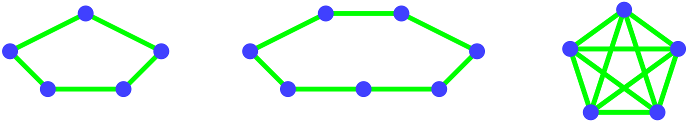

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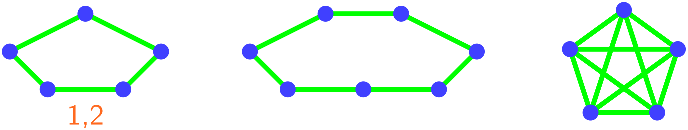

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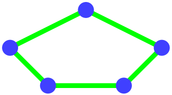

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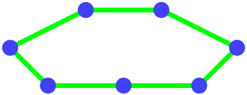

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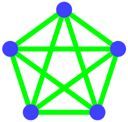

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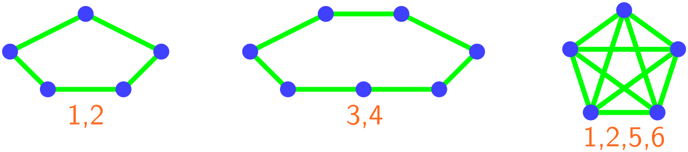

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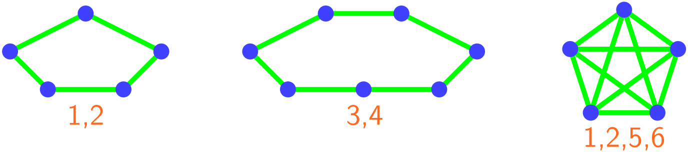

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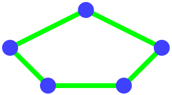

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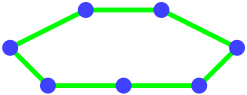

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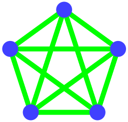

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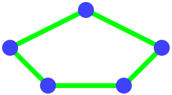

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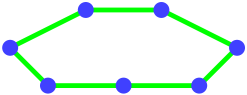

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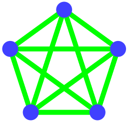

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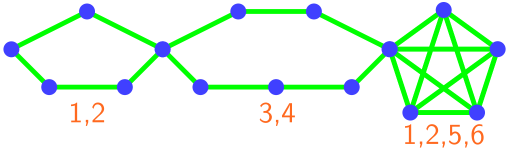

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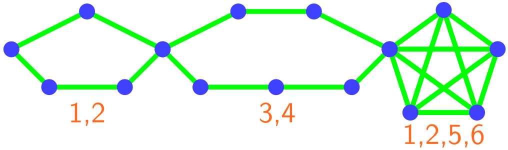

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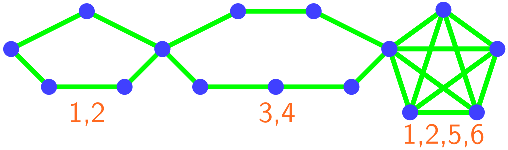

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Big Question: Can we combine Theorems 1 and 3?

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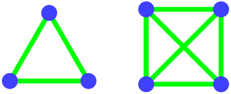

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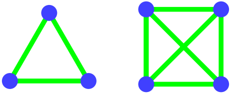

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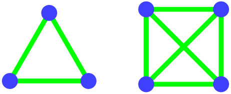

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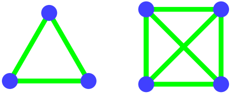

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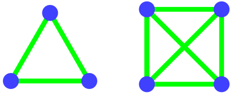

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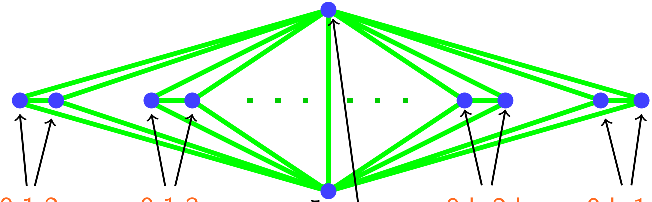

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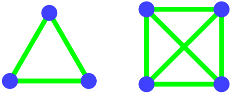

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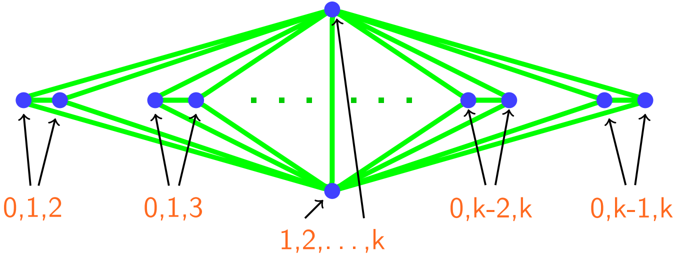

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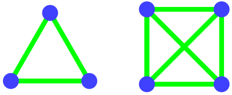

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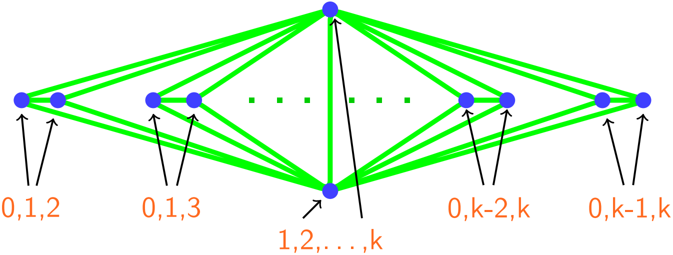

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We have a counterexample when $k = 5$.

Our Main Result

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Proof Sketch of Main Thm:
For each component H of $G[S_k]$, color at most 2 vertices (so that we can finish coloring H later).

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Main Thm: Let G be K_5-minor-free, 3-connected, and not complete. If $k \geq 7$ and $d(S_k) \geq 3$, then G is f-list-colorable when $f(v) = \min\{d(v), k\}$ for all $v \in V(G)$.

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Every K_5-minor-free graph is 5-list-colorable.

Proof Sketch of Main Thm:
For each component H of $G[S_k]$, color at most 2 vertices (so that we can finish coloring H later).
Since $d(S_k) \geq 3$, each $v \in B_k$ loses at most 2 colors.

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Now finish the coloring of each H of $G[S_k]$ (by Theorem 3).

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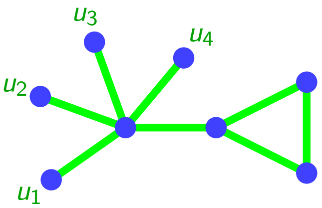

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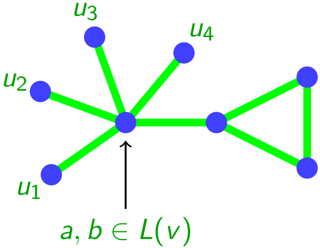

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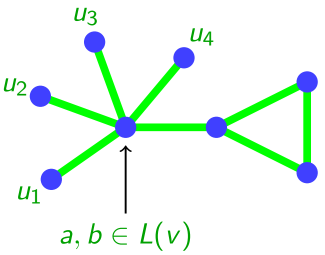

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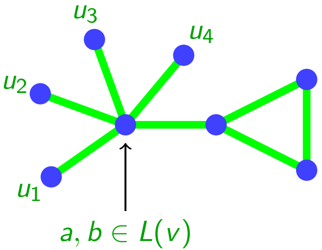

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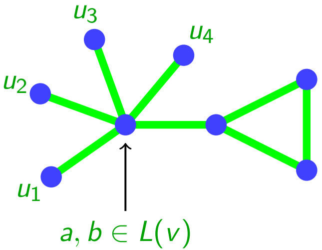

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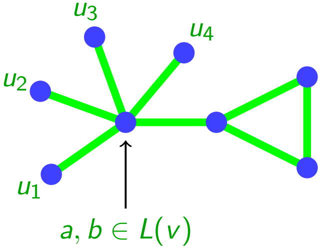

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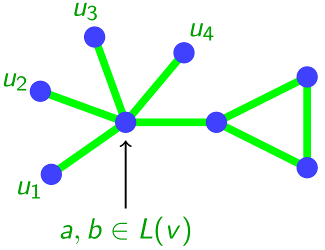

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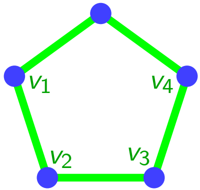

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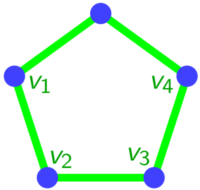

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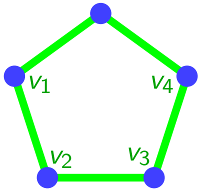

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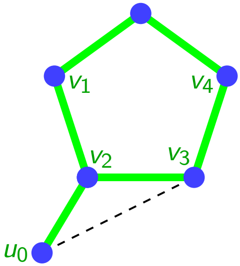

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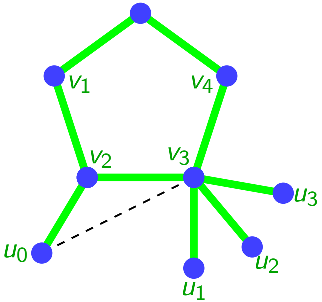

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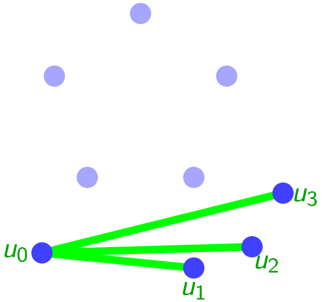

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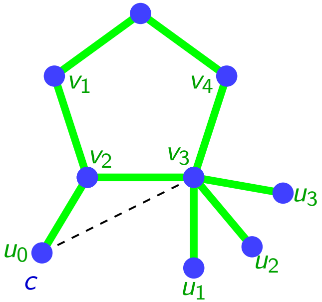

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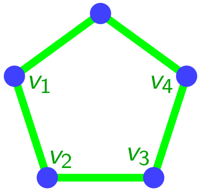

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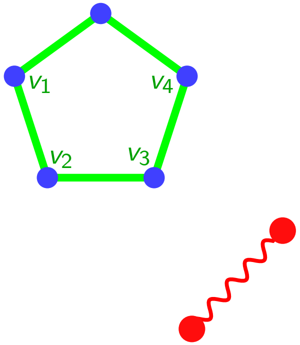

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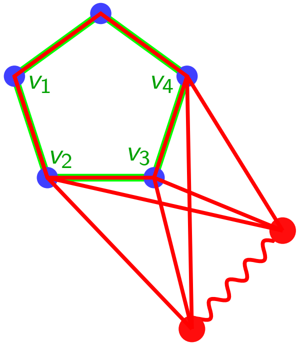

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Tools

Thm 1': [Škrekovski '98]
Every K_5-minor-free graph is 5-list-colorable.

Thm 3: [Vizing '76, Erdős-Rubin-Taylor '79]
Let G be connected and let L be s.t. $|L(v)| \geq d(v)$ for all $v \in V(G)$. If G has no L-coloring, then:

1. $|L(v)| = d(v)$ for every vertex $v \in V(G)$.
2. G is a Gallai tree.
3. Each block B has a list $L(B)$ and $L(v) = \cup_{v \in B} L(B)$.