Reconfiguration of Colorings and List Colorings: Proofs and Conjectures

Daniel W. Cranston
dcransto@gmail.com

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Orsay, France

What is Reconfiguration?

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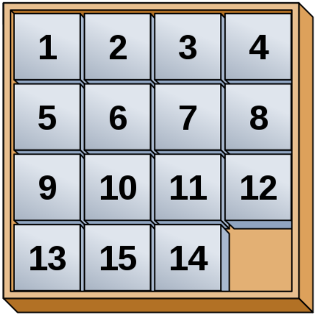

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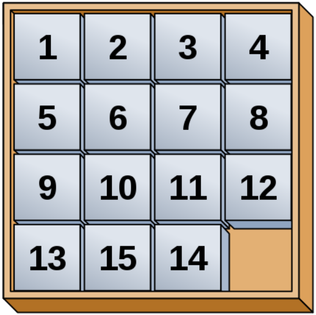

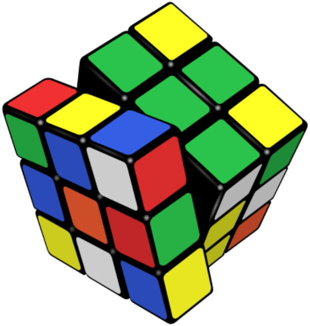

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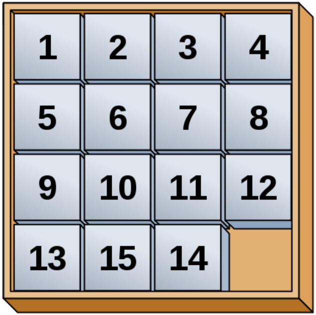

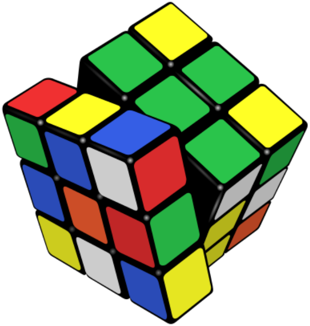

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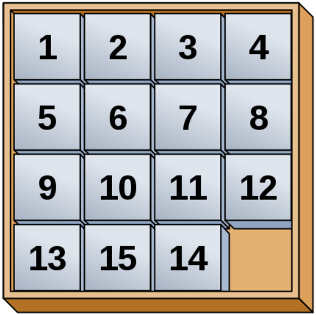

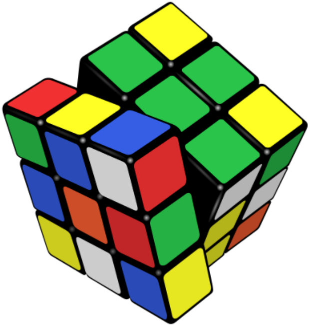

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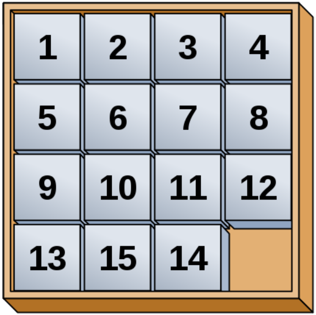

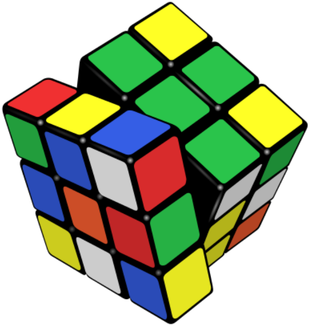

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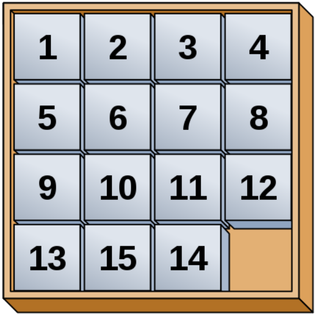

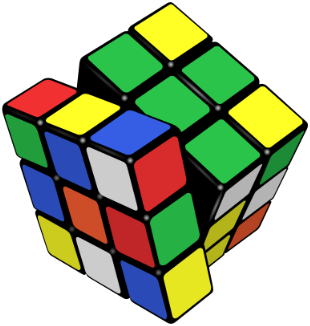

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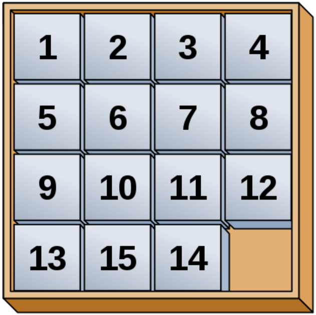

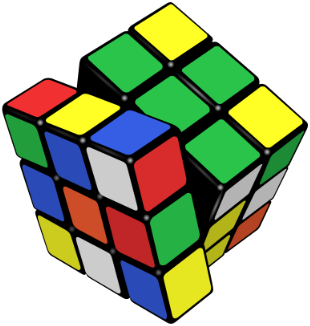

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- If so, how many moves do you need?

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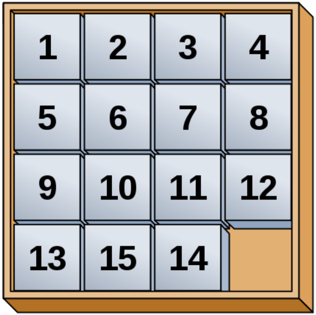

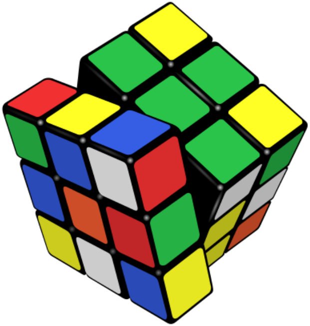

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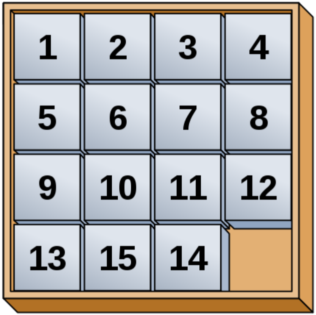

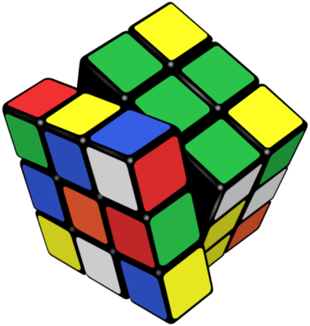

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Move from one instance to another by a sequence of small steps?

- Is it always possible?
- If so, how many moves do you need?
- Can you quickly find a short sequence from one to another?
- Can you quickly sample from all instances (nearly) uniformly?

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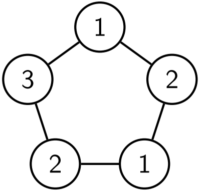

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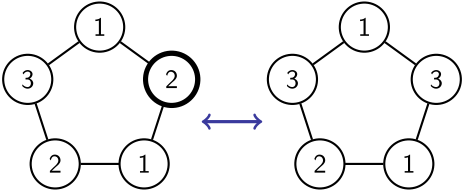

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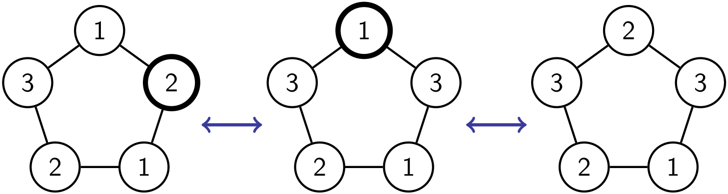

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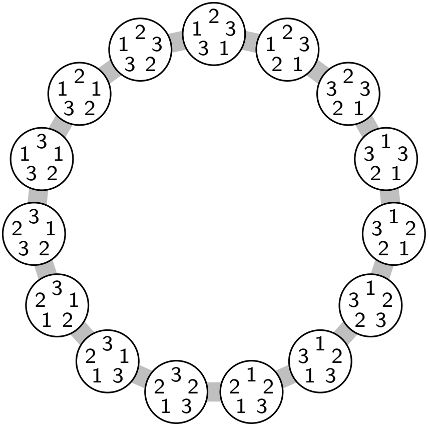

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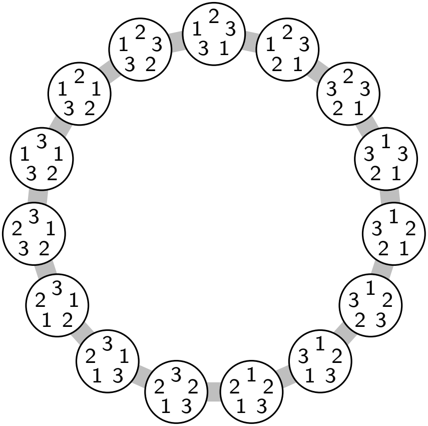

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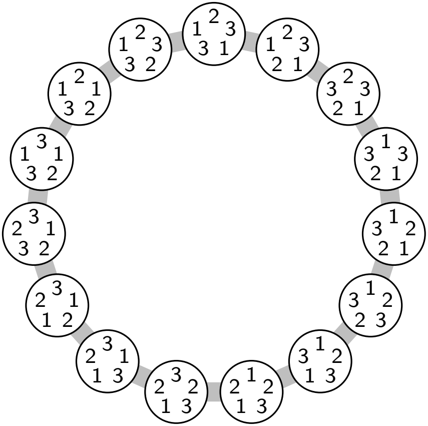

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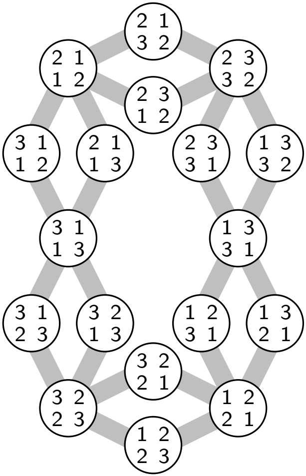

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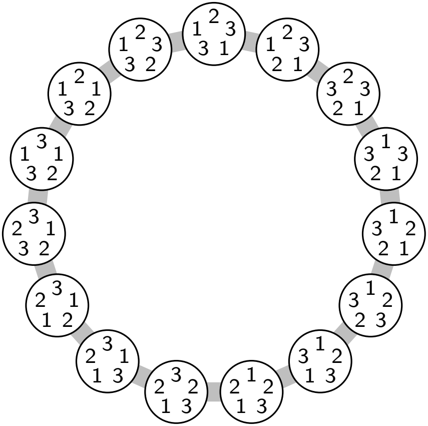

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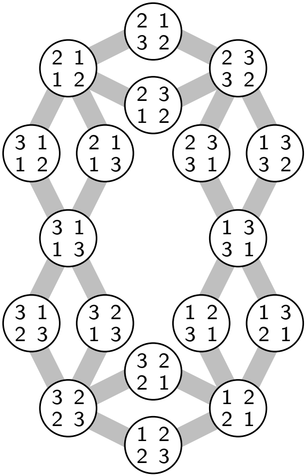

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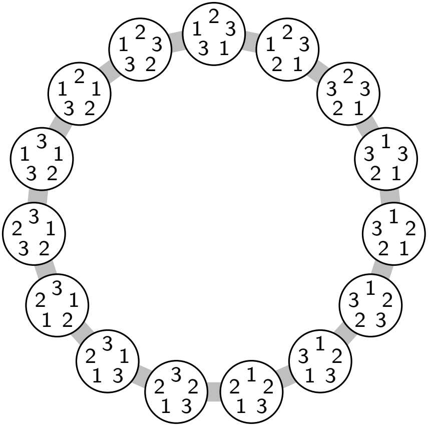

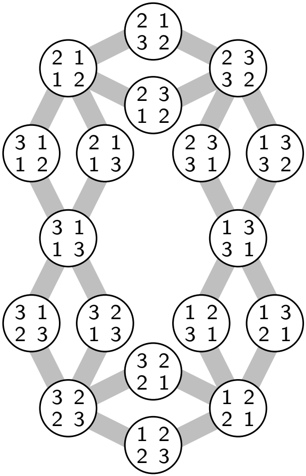

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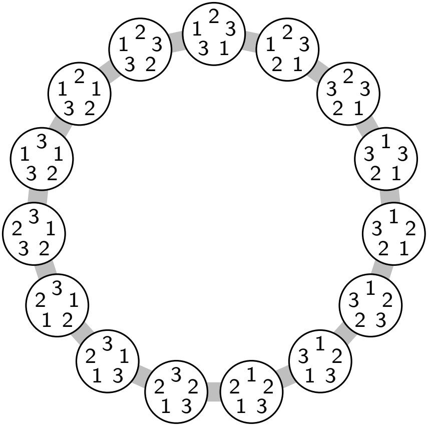

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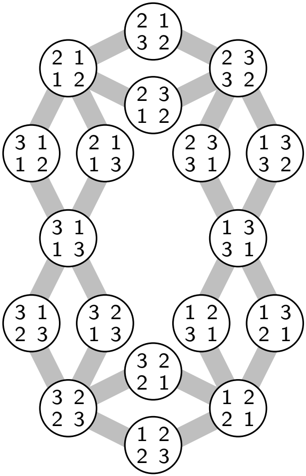
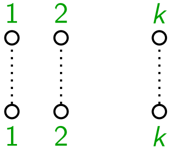


- “Reconfiguration graphs” of 3-colorings of 5-cycle and 4-cycle.
- Is the reconfiguration graph connected? What is its diameter?

Enlightening Examples



$G_k := K_{k,k} - kK_2$

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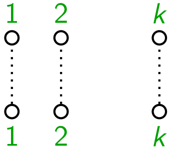

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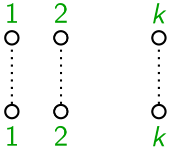

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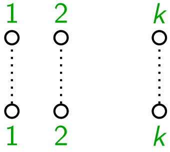

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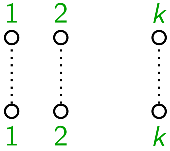

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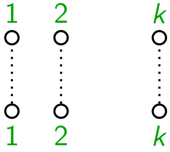

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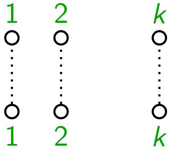

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Bounds on $\mathrm{diam}(\mathcal{C}_k(G))$?

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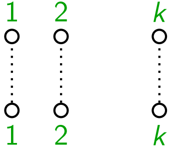

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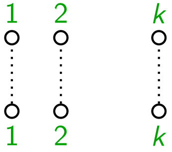

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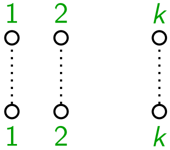

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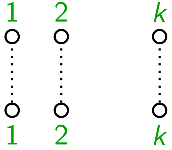

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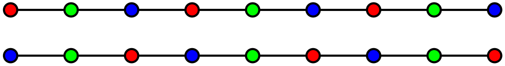

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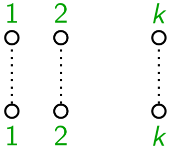

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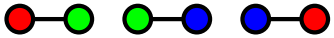
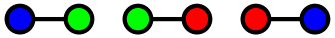
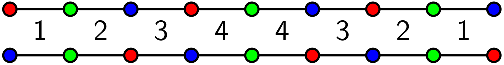
How about "nice" graphs? '$+$':  '$-$': 



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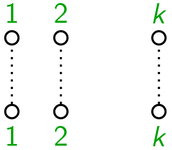

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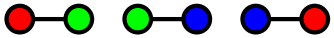
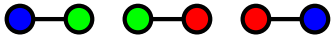
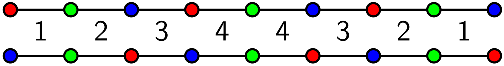
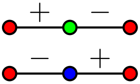
How about "nice" graphs? '+':  '−': 





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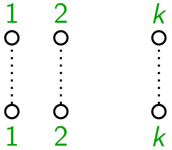

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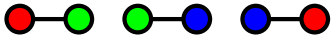
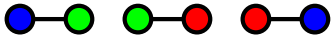
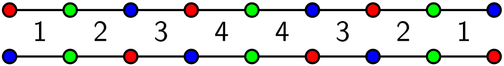
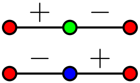
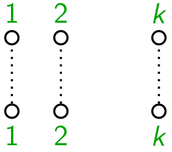
How about "nice" graphs? '+':  '−': 





- So $\mathrm{diam}(\mathcal{C}_3(P_n)) = \Theta(n^2)$

Enlightening Examples



$G_k := K_{k,k} - kK_2$

- $\mathcal{C}_k(G_k)$ is disconnected (frozen colorings)
- $\mathcal{C}_\ell(G_k)$ is connected when $3 \leq \ell < k$.
  Pf: Each part has color repeated, only used on that part. So can reach 2-coloring.
- $\mathcal{C}_\ell(G_k)$ connected when $\ell > k$ (later)

Bounds on $\mathrm{diam}(\mathcal{C}_k(G))$?

- Trivial upper bound: $k^{|G|}$
- Non-trivial lower bound: $2^{\Theta(\sqrt{|G|})}$ (with 4 colors)
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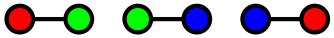
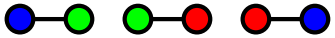
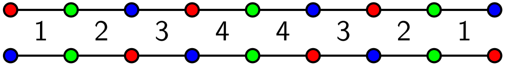
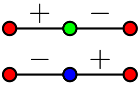
How about "nice" graphs? '+':  '−': 





- So $\mathrm{diam}(\mathcal{C}_3(P_n)) = \Theta(n^2)$ and $\mathrm{diam}(\mathcal{C}_k(P_n \vee K_{k-3})) = \Theta(n^2)$

What is List Coloring Reconfiguration?

- list-assignment L: each vertex v gets allowable colors $L(v)$

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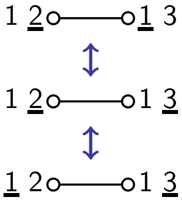

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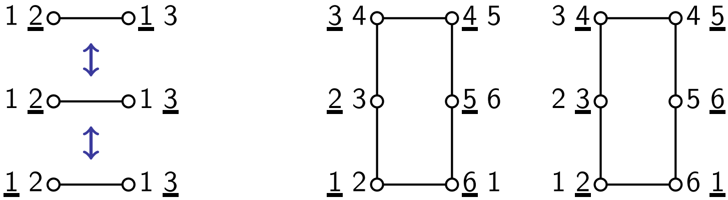

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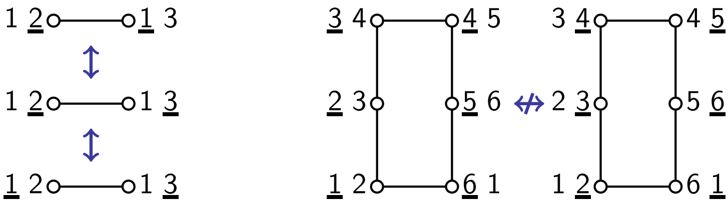

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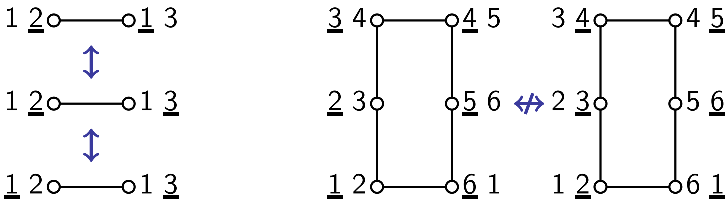

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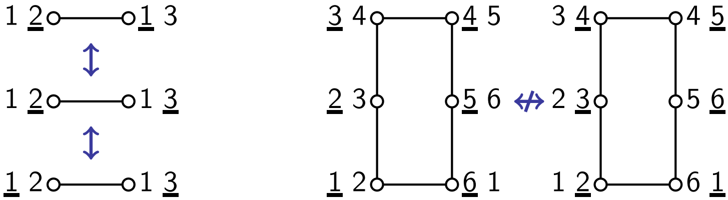

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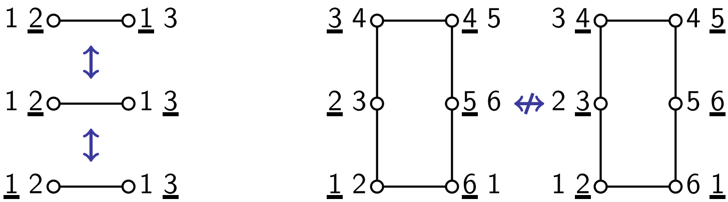

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- Given list-assignment L, can we transform every L-coloring α into every L-coloring β?
- If so, how many steps are needed in the worst case?

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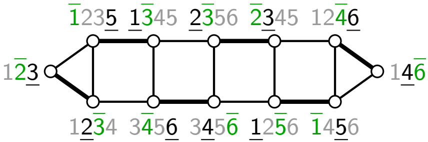

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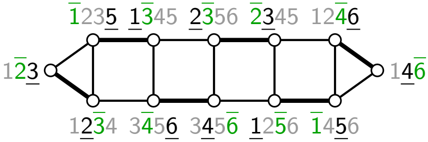

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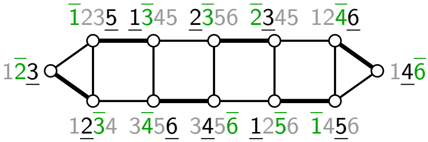

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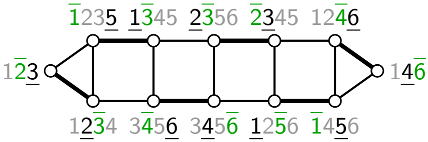

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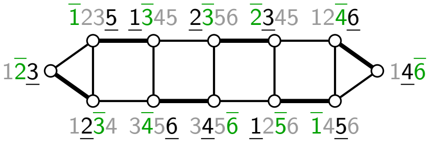

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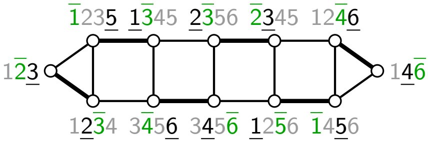

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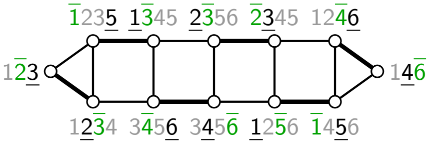

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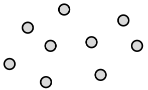

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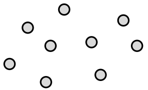

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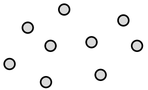

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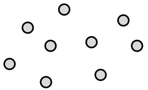

Conj: If $\delta(G) \geq 3$, then we can improve to diameter $O(n)$ in (a) the Key Lem and (b) the Main Thm. What about correspondence?

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Pf: Now a common neighbor y of x_1, x_2 effectively has an extra color.

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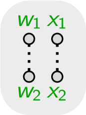
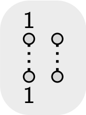
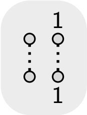




α



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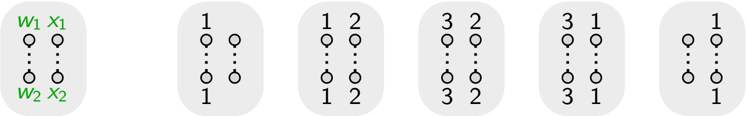

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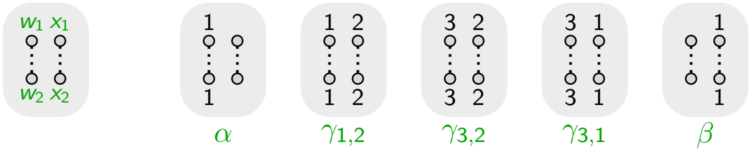

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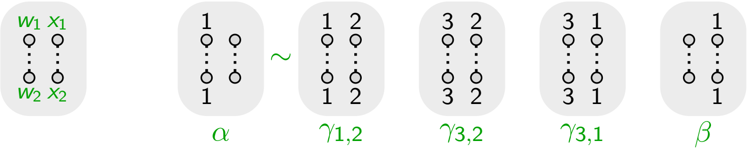

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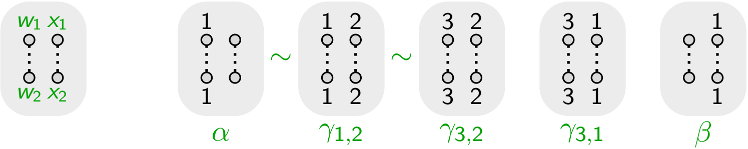

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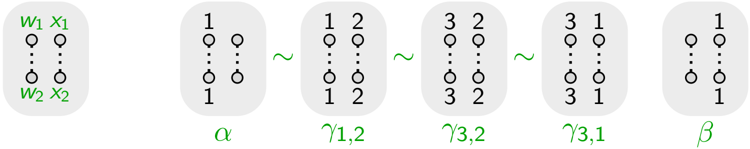

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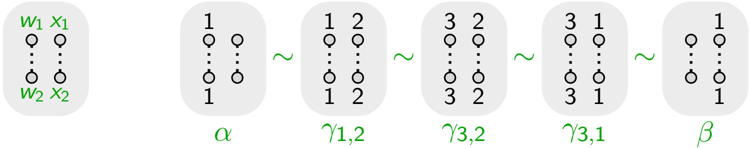

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- $\mathcal{G}$ is planar graphs with girth at least g, for some g.

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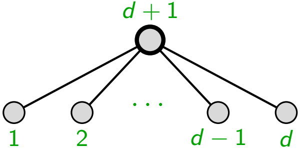

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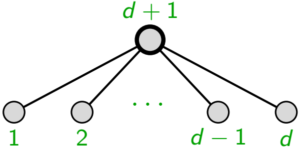

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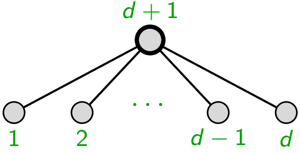

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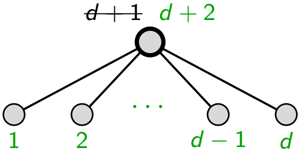

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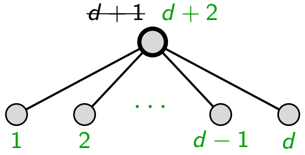

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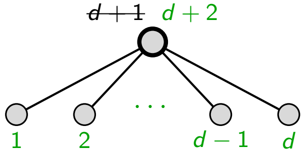

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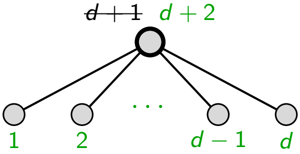

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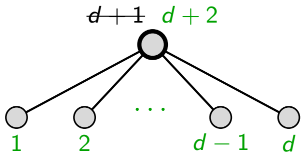

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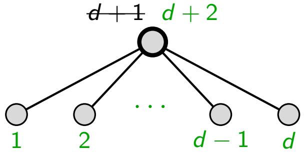

$$c_1, c_2, c_3, \overset{\downarrow}{c_4}, c_5, c_6, c_7, \overset{\downarrow}{c_8}, \ldots$$

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Pf: Like above, but delete many vertices at once.

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- $\mathcal{F}(G, A)$ conn'd when G is 4-edge-conn'd & $|A| \geq 5.3 \times 10^6$.

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Thm: If planar and $|L(v)| \geq 10$, then $\operatorname{diam} \mathcal{C}(G, L) = O(n)$.
Thm: If planar, ∇-free and $|L(v)| \geq 7$, then $\operatorname{diam} \mathcal{C}(G, L) = O(n)$.

Summary

Thm: If $|L(v)| \geq d(v) + 2$, then $\text{diam}\ \mathcal{C}(G, L) \leq n + 2\mu(G)$.
Thm: If $|L(v)| \geq 2d(v) + 1$, then $\text{diam}\ \mathcal{C}(G, L) \leq n + \mu(G)$.
Conj: If $|L(v)| \geq d(v) + 2$, then $\text{diam}\ \mathcal{C}(G, L) \leq n + \mu(G)$.
Correspondence: $\mu(G) \to \tau(G)$.

Thm: If $|L(v)| \geq d(v)+1$ and $\Delta \geq 3$, then $\text{diam}\,\widehat{\mathcal{C}}(G, L) \leq O(n^2)$.
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Conj: Let $d := \text{mad}(G)$. If $|L(v)| \geq \lceil d + 2 \rceil$, then
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Correspondence: Analogues of theorems are true.